Vibration Analysis of Clamped Square Orthotropic Plate

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An approximate method is presented for the determination of the natural frequencies and mode shapes of a square orthotropic plate with all sides clamped. The presentation of the clamped plate's frequency in a form analogous to the corresponding frequency of a simply supported plate is postulated, considering the wave numbers as unknown quantities. These are determined from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems. It is shown, that our method reduces to Bolotin's asymptotic method for large wave numbers. The solution is given for the case of equal coefficients of orthotropy and mode shapes with double symmetry. The results are compared with those known from the literature.

I. Introduction

WIDESPREAD employment of filamentary composite materials in several fields of modern technology has made it desirable to investigate the static and dynamic behavior of structures under the effects of material anisotropy. Analytical and experimental studies of small deflection free vibrations of orthotropic plates have been made by many authors. The most comprehensive review of available results was written by Leissa.[1] An exact solution of the differential equation of a vibrating orthotropic plate is known for the case of a rectangular plate, simply supported along one pair of opposite edges (Levy's problems). The analytical solution of Levy's problems was presented by Huffington and Hoppmann.[2] Numerical results for different Levy's type problems are given in Ref. 1. The solution for the orthotropic plate with all sides simply supported was given by Hearmon.[3] The exact solution for the plate with all sides clamped is unknown. Tomotika's[4] "exact" solution for clamped isotropic plates is not yet generalized for the orthotropic ones. At the same time a considerable number of approximate solutions are available in the literature for several combinations of boundary conditions, including the case of clamped plates. With different degrees of approximation Hearmon,[5] Lekhnitski[6] and many others[1] calculated the fundamental frequency by the method of Rayleigh or Rayleigh-Ritz. Kanazawa and Kawai,[7] using the integral equations technique, calculated the lower frequencies for different combinations of orthotropy coefficients. The asymptotic method, advanced by Bolotin, was realized for the orthotropic plates in Ref. 9. A perturbation method was presented recently by Bauer and Reiss,[11] where the leading term in the perturbation expansion was the solution of "corresponding" isotropic plate. Reference 11 contains new results (using simple approximations) and a list of references dealing with free vibrations of generally orthotropic plates.

The present paper deals with the dynamic analysis of a clamped square orthotropic plate. As a point of departure, the presentation of the frequencies in a form fully analogous to the corresponding frequency of simply supported plate is used. For a simply supported plate, the wave numbers are equal to $m\pi/a$ and $n\pi/a$, respectively,[3] where a denotes the length of the square's side and m and n are positive integers which determine the number of mode shape. In our analysis, for a clampled plate the wave numbers are presented in the form $p\pi/a$ and $q\pi/a$ where the pair of real quantities p and q are to be found from the solution of two supplementary eigenvalue problems. The integer parts of p and q represent frequency numbers. The mode shape is presented as a product of eigenfunctions, corresponding to these supplementary problems.

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II. Analytical Formulation

Free small amplitude vibrations of a thin, elastic orthotropic plate are governed by the linear partial differential equation

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho \frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where D_x and D_y are flexural rigidities and are defined by

$$D_x = \frac{E_x h^3}{12(1-v_x v_y)}, \qquad D_y = \frac{E_y h^3}{12(1-v_x v_y)}, \qquad D_k = \frac{Gh^3}{12}$$

$$H = D_x v_y + 2D_k$$

E_x and E_y are Young's moduli along the X and Y axes, respectively, G is the rigidity modulus, v_x and v_y are Poisson's ratios for the material where $E_x v_y = E_y v_x$, ρ is the mass density per unit area of the plate, h is the plate thickness, t is the time. It is assumed in Eq. (1) that the principal elastic axes of the material are parallel to the plate edges.

When free vibrations are assumed, the motion is expressed as[12]

$$w(x, y, t) = W \sin \omega t \tag{2}$$

where ω is the circular frequency and W is a function of the position coordinates only. Substituting Eq. (2) into Eq. (1) yields

$$D_x \frac{\partial^4 W}{\partial x^4} + 2H \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} - \rho\omega^2 W = 0 \tag{3}$$

For a plate with all sides simply supported the boundary conditions are

$$W = \nabla^2 W = 0 \tag{4}$$

where ∇^2 is the Laplacian operator.

For a plate with all sides clamped the boundary conditions are

$$W = \partial W/\partial \tau = 0 \tag{5}$$

where τ denotes the direction normal to the contour.

For the problems (2) and (4) it is seen[3] that

$$W_{mn} = A_{mn} \sin(m\pi x/a) \sin(n\pi y/a) \tag{6}$$

satisfies the boundary conditions where A_{mn} is an amplitude coefficient determined from the initial conditions of the problem and m and n are positive integers. Substituting Eq. (6) into Eq. (2) gives the frequency

$$\omega = \left(\frac{H}{\rho}\right)^{1/2} \left[\frac{D_x}{H}\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{a}\right)^2 + \frac{D_y}{H}\left(\frac{n\pi}{a}\right)^4\right]^{1/2} \tag{7}$$

Our purpose is to find the solution of Eq. (2) with the boundary conditions (5), i.e., the eigenfrequencies and the mode shapes of a clamped orthotropic square plate. We shall postulate the following presentation of the eigenfrequency analogous to the case of a plate with all sides simply supported [Eq. (7)]

$$\omega = \left(\frac{H}{\rho}\right)^{1/2}\left[\frac{D_x}{H}\left(\frac{p\pi}{a}\right)^4 + 2\left(\frac{p\pi}{a}\right)^2\left(\frac{q\pi}{a}\right)^2 + \frac{D_y}{H}\left(\frac{q\pi}{a}\right)^4\right]^{1/2} \tag{8}$$

where p and q are real numbers which are to be found.

For the determination of p and q we first consider two auxiliary problems of Levy's type.

IIa. First Auxiliary Problem

Let us consider the following class of solutions for Eq. (3):

$$W(x, y) = Y\sin(p\pi x/a) \tag{9}$$

satisfying the boundary conditions

$$Y = dY/dy = 0 \quad \text{at} \quad y = 0, a \tag{10}$$

Substituting Eq. (9) into Eq. (2) results in an ordinary differential equation for $Y(y)$

$$\frac{d^4Y}{dy^4} - 2\frac{p^2\pi^2}{a^2}\frac{H}{D_y}\frac{d^2Y}{dy^2} + \left(\frac{p^4\pi^4}{a^4}\frac{D_x}{D_y} - \frac{\rho\omega^2}{D_y}\right)Y = 0$$

or, given the postulating Eq. (8), we obtain

$$\frac{d^4Y}{dy^4} - 2\frac{p^2\pi^2}{a^2}\frac{D}{D_y}\frac{d^2Y}{dy^2} - \frac{q^2\pi^2}{a^2}\left(2\frac{H}{D_y}\frac{p^2\pi^2}{a^2} + \frac{q^2\pi^2}{a^2}\right)Y = 0 \tag{11}$$

The corresponding characteristic equation

$$\lambda^4 - 2\frac{p^2\pi^2}{a^2}\frac{H}{D_y}\lambda^2 - \frac{q^2\pi^2}{a^2}\left(2\frac{H}{D_y}\frac{p^2\pi^2}{a^2} + \frac{q^2\pi^2}{a^2}\right) = 0$$

has two imaginary and two real roots, namely

$$\lambda_{1,2} = \pm i\frac{q\pi}{a}, \qquad \lambda_{3,4} = \pm\frac{\pi}{a}\kappa \tag{12}$$

where

$$\kappa = [2p^2(H/D_y) + q^2]^{1/2}$$

The general solution of Eq. (11) can be represented as

$$Y(y) = A_1\cosh(\kappa\pi\eta) + A_2\sinh(\kappa\pi\eta) + A_3\cos(q\pi\eta) + A_4\sin(q\pi\eta) \tag{13}$$

where A_j are constants of integration and $\eta \equiv y/a$.

When Eq. (13) is substituted into Eqs. (10) the existence of a nontrivial solution yields the characteristics determinant

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & \kappa & 0 & q \\ C_2 & S_2 & c_2 & s_2 \\ \kappa S_2 & \kappa C_2 & -qs_2 & qc_2 \end{vmatrix} = 0$$

where

$$c_2 = \cos q\pi, \quad s_2 = \sin q\pi, \quad C_2 = \cosh\kappa\pi, \quad S_2 = \sinh\kappa\pi \tag{15}$$

Expanding Eq. (14) and substituting Eq. (12) into the result gives

$$1 - c_2C_2 + \frac{H}{D_y}\left(2\frac{H}{D_y}\frac{q^2}{p^2} + \frac{q^4}{p^4}\right)^{-1/2} s_2S_2 = 0 \tag{16}$$

IIb. Second Auxiliary Problem

Now consider another class of solutions of Eq. (3)

$$W(x, y) = X\sin(q\pi y/a) \tag{17}$$

satisfying the boundary conditions

$$X = dX/dx = 0 \quad \text{at} \quad x = 0, a \tag{18}$$

The solution of this problem could be obtained from the solution of the first auxiliary problem by substitution $D_y \to D_x$, $p \to q$, $q \to p$. We obtain the following characteristic equation

$$1 - c_1C_1 + \frac{H}{D_x}\left(2\frac{H}{D_x}\frac{p^2}{q^2} + \frac{p^4}{q^4}\right)^{-1/2} s_1S_1 = 0 \tag{19}$$

where

$$c_1 = \cos p\pi, \quad s_1 = \sin p\pi, \quad C_1 = \cosh\theta\pi, \quad S_2 = \sinh\theta\pi$$
$$\theta = [2q^2(H/D_x) + p^2]^{1/2}, \qquad \xi \equiv x/a \tag{20}$$

III. Frequencies and Mode Shapes

Equations (16) and (19) being transcendental in nature, have an infinite number of roots. The unknown quantities p and q are calculated from the solution of these equations. Substituting p and q into the postulated Eq. (8), we obtain the eigenfrequencies. The integer parts $[p]$ and $[q]$ represent the numbers of the eigenfrequency. The mode shapes are determined as the product

$$W_{[p],[q]} = X(x)Y(y) \tag{21}$$

where

$$X(x) = \cosh\theta\pi\xi - \cos p\pi\xi + \left(\frac{S_1 - \theta p^{-1}s_1}{C_1 - c_1}\right)^{-1}\left(\sinh\theta\pi\xi - \frac{\theta}{p}\sin p\pi\xi\right) \tag{22}$$

$$Y(y) = \cosh\kappa\pi\eta - \cos q\pi\eta + \left(\frac{S_2 - \kappa q^{-1}s_2}{C_2 - c_2}\right)^{-1}\left(\sinh\kappa\pi\eta - \frac{\kappa}{q}\sin q\pi\eta\right)$$

It should be noted that Eq. (21) satisfies all the boundary conditions, but not the field Eq. (3) and thus contributes an approximate solution of the problem.

We observe that in the particular case, $D_x = D_y$, the system (16) and (19) has the solutions of the following type (p, q) and (q, p). To the two solutions of this type correspond two mode shapes. As can be seen from Eq. (8), these two mode shapes have the same frequency. They exist simultaneously, their initial amplitudes depending upon the initial conditions. This fact is also true for an isotropic plate (see Ref. 1, p. 45). In the case $D_x = D_y$ and $p = q$, the system (16) and (19) reduces to one transcendental equation

$$1 - cC + (H/D)(2H/D + 1)^{-1/2}sS = 0 \tag{23}$$

where, as it follows from Eqs. (12) and (20)

$$c_1 = c_2 = c, \quad s_1 = s_2 = s, \quad C_1 = C_2 = C, \quad S_1 = S_2 = S$$
$$D_x = D_y = D, \quad \kappa = \theta = p(2H/D + 1)^{1/2}$$

IV. Comparison with Bolotin's Asymptotic Method

Consider a case where $p \gg 1$ and $q \gg 1$. In this case it is possible to simplify Eqs. (16) and (19). Neglecting unity compared with the values of hyperbolic functions and assuming that $\tanh x \sim 1$, we obtain

$$\tan p\pi = \frac{H}{D_y}\left(2\frac{H}{D_y}\frac{q^2}{p^2} + \frac{q^4}{p^4}\right)^{1/2}$$
$$\tan q\pi = \frac{H}{D_x}\left(2\frac{H}{D_x}\frac{p^2}{q^2} + \frac{p^4}{q^4}\right)^{1/2} \tag{24}$$

Equations (24) could be obtained by using Bolotin's asymptotic method.[8] According to this method the asymptotic solution for the natural mode is expressed by a sum of the internal solution and correction solutions which are called the dynamic boundary effects. Each expression satisfies the differential equation and the conditions on *only* one boundary of the plate. The number of these expressions is equal to the number of boundaries. The obtained expressions for each pair of opposite edges must be "matched up." The condition for "matching up" these expressions are equations of the same type as Eqs. (24). The "matching up" process is only *approximate*.

In our analysis the "matching up" of solutions was superfluous, hence the functions $X(x)$ and $Y(y)$ satisfy the boundary conditions on both opposite edges. The method presented herein could be considered, in some sense, as an extension of Bolotin's asymptotic method.

From Eq. (23) Bolotin's type simple "exact" solution could be obtained. Neglecting unity and replacing $C \to S$, we arrive at the equation

$$\tan p\pi = [1 + (2/\alpha)]^{1/2}, \quad \alpha = D_x/H, \quad (\beta = D_y/H)$$

Hence

$$p = q = m + (1/\pi)\tan^{-1}[1 + (2/\alpha)]$$

Table 1 First ten frequency coefficients ω ($\alpha = \beta = 2$)

m	n	p	q	$\bar{\omega}$
1	1	1.39306	1.39306	22.59586
1	2	1.27437	2.46221	98.47318
2	1	2.46221	1.27437	98.47318
2	2	2.39181	2.39181	196.36289
1	3	1.20060	3.48214	333.15638
3	1	3.48214	1.20060	333.15638
2	3	2.32494	3.44049	466.62778
3	2	3.44049	2.32494	466.62778
3	3	3.39183	3.39183	794.11954
1	4	1.15550	4.48979	870.10579
4	1	4.48979	1.15550	870.10579
2	4	2.27104	4.46344	1052.49998
4	2	4.46344	2.27104	1052.49998
3	4	3.34532	4.42875	1458.89534
4	3	4.42875	3.34532	1458.89534
1	5	1.12617	5.49345	1901.18475
5	1	5.49345	1.12617	1901.18475

and for frequency

$$\omega_{[p]}^2 \equiv \omega_{[p][p]}^2 = 2(1+\alpha)\frac{H}{\rho}\frac{\pi^4}{\alpha^4}\left[m+\frac{1}{\pi}\tan^{-1}\left(1+\frac{2}{\alpha}\right)\right]^2 \quad (25)$$

For $\alpha = 1$, Bolotin's[1,8] general formula for the vibrating frequency of isotropic square plates when $m = n = [p]$ yields

$$\omega_m^2 = 4(D/\pi)(\pi^4/a^4)(m+\tfrac{1}{3})^2 \quad (26)$$

V. Numerical Results

The numerical examples were worked out on an IBM 370/165 at the Technion. After finding the region in which the roots of the Eqs. (16) and (19) occurred, we applied Newton's method to obtain good approximate values of the roots. The double precision arithmetic was used where necessary. In the plane p, q, the square defined by the lines $p = m$, $p = m+1$, and $q = n$, $q = n+1$, where m and n are any positive integers, there is only one root of Eqs. (16) and (19). This allows us to find the frequency for any given number m, n. In particular, the lowest frequency corresponds to the solution of Eqs. (16) and (19), satisfying the following inequalities—$1 < p < 2$ and $1 < q < 2$.

In Table 1 are presented the calculated results of the frequency coefficient $\bar{\omega} = a^2\gamma/\pi^2$, where $\gamma = \omega(\rho/H)^{1/2}$ for the first ten frequencies when $D_x/H = D_y/H = D/H = 2$. Repeated frequencies are considered as one. For the lowest frequency coefficient Kanazawa and Kawai[7] gave the value 24.251 so that the percentage difference is less than 7%. In Table 2 a comparison of the results of Hearmon,[5] Lekhnitski,[6] and Kanazawa and Kawai[7] is presented for the lowest frequencies. Frequency coefficient $\bar{\omega}$ according to Lekhnitski[6] is

$$\bar{\omega} = 504^{1/2}\pi^{-2}(\alpha+\beta+0.571)^{1/2}$$

and according to Hearmon[5]

$$\bar{\omega} = 501^{1/2}\pi^{-2}(\alpha+\beta+0.605)^{1/2}$$

where α and β denote the orthotropy coefficients.

In Table 3 our results are compared with those of Bolotin et al.[9] The case which they computed was $\alpha = 1$, $\beta = 10$ for the region $\bar{\omega} < 71.371$. The percentage difference is of the order 1% except for the case when $m = 4$, $n = 3$, which is, probably, a typographical error in Bolotin's paper. The calculation according to the equations given by Bolotin in $\bar{\omega} = 47.343$ differs from our results only by 0.185%. From Table 3 it may be seen that some frequencies were not found by Bolotin et al.[9]

Table 2 Estimates of the coefficient $\bar{\omega}(\alpha,\beta)$ for the fundamental frequency[a]

β	Methods	$\alpha = \frac{1}{3}$	$\alpha = \frac{1}{2}$	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$
$\frac{1}{3}$	Present	6.105	6.935	9.449	14.540	19.659
	Ref. 6	6.404	7.266	9.853	15.027	20.201
	Ref. 5	6.541	7.398	9.969	15.113	20.256
	Ref. 7	6.434	7.341	10.016	15.355	20.638
$\frac{1}{2}$	Present		7.750	10.235	15.300	20.408
	Ref. 6		8.128	10.715	15.890	21.064
	Ref. 5		8.255	10.827	15.970	21.113
	Ref. 7		8.249	10.927	16.242	21.553
1	Present			12.658	17.661	22.740
	Ref. 6			13.302	18.477	23.651
	Ref. 5			13.398	18.541	23.685
	Ref. 7			13.608	18.928	24.232
2	Present				22.596	27.640
	Ref. 6				23.651	28.825
	Ref. 5				23.685	28.828
	Ref. 7				24.251	29.561
3	Present					32.665
	Ref. 6					33.999
	Ref. 5					33.971
	Ref. 7					34.873

[a] $\bar{\omega}(\alpha,\beta) = \bar{\omega}(\beta,\alpha)$.

Table 3 Comparison with Bolotin's asymptotic method ($\alpha = 1$, $\beta = 10$, $\bar{\omega} \leqq 71{,}371$)

				Frequency coefficient		
m	n	p	q	Bolotin's method	Present method	Percent diff.
1	1	1.310713	1.481124	7.570	7.655940	1.135
2	1	2.416360	1.431006	9.971	9.996936	0.270
3	1	3.456417	1.374230	14.936	14.950399	0.096
1	2	1.198223	2.492598	20.137	20.147493	0.052
2	2	2.321523	2.474074	20.587	21.672486	0.396
4	1	4.474458	1.320842	22.249	22.385843	0.615
3	2	3.388547	2.448549	25.060	25.079150	0.076
4	2	4.426115	2.419281	30.898	30.914035	0.052
5	1	5.483661	1.275270	...	32.070147	...
1	3	1.140763	3.496657	39.064	39.094960	0.079
5	2	5.448452	2.388815	...	39.315009	...
2	3	2.250508	3.487277	40.300	40.345538	0.113
3	3	3.325278	3.473246	42.965	42.945491	0.045
6	1	6.488816	1.238092	...	43.879246	...
4	3	4.374447	3.455904	42.265	47.430835	12.223
6	2	6.462488	2.358953	...	50.186322	...
5	3	5.407109	3.436410	...	54.219994	...
7	1	7.491931	1.208207	...	57.755059	...
7	2	7.471734	2.330825	...	63.390573	...
6	3	6.429354	3.415748	...	63.518162	...
1	4	1.108207	4.498079	64.281	64.380374	0.155
2	4	2.201427	4.492535	65.450	65.517881	0.104
3	4	3.273813	4.483883	67.683	67.735035	0.077
4	4	4.327224	4.472723	71.371	71.427666	0.079

VI. Conclusions

New analytical results on the free vibration of clamped orthotropic plates have been presented. We have shown that the use of "postulate" (8) and of two Levy's type solutions to approximate natural frequencies and mode shapes of vibrating clamped square orthotropic plates leads to very accurate results. This approximation is more convenient than the method[8] because it provides a very simple way to derive the transcendental Eqs. (16) and (19) for the unknown wave numbers and leads to the determination of mode shapes for the entire region of the plate. These mode shapes are important when studying many forced and random vibration problems. The essential advantage of the presented method is the possibility of finding the frequency and the mode shape for any given pair of mode numbers.

References

[1] Leissa, A. W., *Vibration of Plates*, NASA SP-160, 1969.

[2] Huffington, N. J., Jr. and Hoppmann, W. H., "On Transverse Vibrations of Rectangular Orthotropic Plates," *Journal of Applied Mechanics*, Vol. 25, No. 3, 1958, pp. 389–395.

[3] Hearmon, R. F. S., "The Fundamental Frequency of Vibration of Rectangular Wood and Plywood Plates," *Proceedings of the Physical Society, London*, Vol. 58, 1946, pp. 78–92.

[4] Tomotika, S., "On the Transverse Vibration of a Square Plate Clamped at Four Edges," *Philosophical Magazine*, Vol. 21, Ser. 7, No. 142, 1936, pp. 745–760.

[5] Hearmon, R. F. S., "The Frequency of Flexural Vibration of Rectangular Orthotropic Plates with Clamped or Supported Edges," *Journal of Applied Mechanics*, Vol. 26, Ser. E, No. 4, 1959, pp. 537–540.

[6] Lekhnitski, S. T., *Anisotropic Plates*, American Iron and Steel Institute, New York, 1956.

[7] Kanazawa, T. and Kawai, T., "On the Lateral Vibration of Anisotropic Rectangular Plates (Studied by the Integral Equations)," *Proceedings of 2nd Japan National Congress for Applied Mechanics*, 1952, pp. 333–338.

[8] Bolotin, V. V., "The Edge Effect in the Oscillations of Elastic Shells," *Journal of Applied Mathematics and Mechanics*, Vol. 24, No. 5, 1960, pp. 1257–1272.

[9] Bolotin, V. V., Makaroff, B. P., Mishenkoff, G. V., and Shveiko, Yu. Yu., "Asymptotic Method of Investigating the Natural Frequency Spectrum of Elastic Plates," *Rascheti na Prochnost*, Vol. 6, Mashgiz, Moscow, 1960, pp. 231–253 (in Russian).

[10] Bauer, L. and Reiss, E. L., "Flexural Vibrations of Clamped Orthotropic Plates," *Journal of the Acoustical Society of America*, Vol. 53, No. 5, 1973, pp. 1360–1364.

[11] Maurizi, M. J. and Laura, P. A., "Vibration Analysis of Clamped, Rectangular Plates of Generalized Orthotropy," *Journal of Sound and Vibration*, Vol. 26, No. 3, 1973, pp. 299–305.

[12] Warburton, C. B., *The Dynamical Behavior of Structures*, Pergamon Press, New York, 1964.

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On Multiple-Shaker Resonance Testing

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A simulation study is carried out to explore the use of vector response data from single-shaker modal survey tests in tuning the shaker inputs for multiple-shaker tests. A modal purity criterion based on phase coherence, i.e., displacements should lag excitation by 90°, is defined. A "test natural frequency" is defined as any excitation frequency which produces, at all excitation stations, response in quadrature with the excitation forces. A "test natural mode" is defined as the quadrature response at all measurement stations when the system is excited at a test natural frequency. Calculations based on single-shaker response data determine "test natural frequencies," "test natural modes," and the required shaker force ratios for multiple-shaker tests. These frequencies and modes are compared with exact ones for two nine-degree-of-freedom damped systems, one having two frequencies very closely spaced. It is shown that both acceptable and spurious test natural frequencies are obtained, but that acceptable ones persist (i.e., occur for most possible shaker combinations). "Test natural modes" corresponding to spurious frequencies have poor phase coherence at nonexcited stations and can thus be readily identified. Acceptable mode shape information can be obtained even when frequencies are closely spaced.

Nomenclature

$[\bar{B}]$ = admittance matrix for all n coordinates
$[\bar{B}_*]$ = admittance matrix for p coordinates
$[C]$ = damping coefficient matrix
$\{F\}$ = input force vector at n stations
$\{F_*\}$ = input force vector at p stations
$[K]$ = stiffness matrix
$[M]$ = mass matrix
n = number of degrees of freedom, total number of displacement measurement stations
$\{p\}$ = principal coordinates
p = number of shakers used
$\{\bar{u}\}$ = harmonic response = $\{\bar{U}\}\, e^{i\omega t}$
$\{v\}$ = original displacement coordinates
$\{V\}$ = response amplitude vector
$[V]$ = modal matrix
$[\gamma]$ = damping matrix in principal coordinates
ε = allowable phase error
θ = phase lag angle
$[\kappa]$ = stiffness matrix in principal coordinates
$[\mu]$ = mass matrix in principal coordinates
$\{\Phi\}$ = force vector in principal coordinates
ω, ω_r = excitation frequency (rad/sec), rth natural frequency

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Introduction

METHODS for experimentally determining the dynamical characteristics of structures using resonance, or modal survey, testing may be classified as single-shaker or multiple-shaker methods. The latter category includes the possibility of using information from one or more single-shaker tests in setting up multiple-shaker tests. The purpose of resonance testing is to determine information on one or more of the following dynamical characteristics of the system: natural frequencies, mode shapes, damping factors, and generalized masses.

Kennedy and Pancu[1] suggested that certain characteristics of vector response plots obtained with single-shaker excitation could be used in determining the natural frequencies and damping factors of systems. Bishop and Gladwell[2] and Pendered[3] assessed the accuracy of several single-shaker techniques, including the method of Kennedy and Pancu. It may be said that, if properly interpreted, natural frequencies obtained by the method of Kennedy and Pancu are generally acceptable, but mode-shapes results are frequently unacceptable.